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Universal R-matrix and graded Hopf algebra structure of $U_q(\widehat{gl}(2|2))$

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Abstract. An explicit formula for the universal R-matrix of $U_q(\widehat{gl}(2|2))$ is given using Drinfeld's basis. Its generators are chosen according to a non-standard set of positive simple roots. The expression implies an extension of the standard graded Hopf algebra $U_q(\widehat{sl}(2|2))$ defined in terms of its Chevalley generators to $U_q(\widehat{gl}(2|2))$. In addition to a four-dimensional vector representation, an infinite-dimensional representation of $U_q(gl(2|2))$ suitable for the description of a model related to the integer quantum Hall transition is considered.

1. Introduction

A number of models associated with affine superalgebras have attracted recent attention in context with two-dimensional disordered systems. Most investigations have focused on electronic systems in the presence of a random Abelian or non-Abelian vector field [1] which are accessible by analytic treatment due to an underlying structure encoded by the current superalgebra $\widehat{gl}(n|n)$. Before this, a detailed study of the Wess–Zumino–Witten model on the supergroup $Gl(1|1)$ had been presented in [2]. Furthermore, Dirac fermions with a random mass have been discussed both in relation to the random bond Ising model [3] and several aspects of the integer quantum Hall transition [4]. For a Gaussian distributed mass, averaging over the disorder yields a Gross–Neveu model associated with $osp(2n|2n)$ which can be viewed as an integrable perturbation of the conformally invariant system characterized by the $\widehat{osp}(2n|2n)$-current superalgebra [5]. The corresponding formulation in terms of a massless scattering matrix was considered in [6].

A very recent study proposes the construction of an integrable vertex model based on a particular module of $gl(2|2)$ to the aim of describing the integer quantum Hall transition [7]. The appropriate affine algebra is expected to involve a graded Yangian structure related to $gl(2|2)$. Another approach consists in studying the corresponding vertex models related to the quantum affine superalgebra $U_q(\widehat{gl}(2|2))$ and to extract results for the $gl(2|2)$-invariant case from the limit $q \to -1$. This limiting procedure has been applied previously to obtain correlation functions for the XXX spin chain from those of the XXZ chain calculated by two different methods [8–10].

So far, studies of integrable vertex models associated with a Lie (super)algebra consider systems with statistical variables taking values in its finite-dimensional modules. In contrast, the description of disordered problems requires constructions taking into account infinite-dimensional modules of $gl(n|n)$ or $osp(2n|2n)$ [7, 11].

Trigonometric solutions of the graded Yang–Baxter equation related to the vector representations of $sl(n|m)$ and $osp(2n|m)$ were constructed in [12]. As emphasized in [13], in the case $n = m$ the non-simple Lie superalgebra $gl(n|n)$ rather than $sl(n|n)$ has to be considered. The irreducible integrable highest-weight representations of affine quantum superalgebras have been classified [14]. The evaluation homomorphism established for $U_q\big(\widehat{sl}(n)\big)$ [15] can be generalized to equip any $U_q\big(sl(n|m)\big)$ module with a $U_q'\big(\widehat{sl}(n|m)\big)$ structure [16]. A method developed for non-critical systems invariant under an affine quantum algebra [8] provides expressions for the correlators in terms of vertex operators [17]. Their evaluation is achieved making use of a representation of the current algebra and its highest weight modules by means of deformed bosonic oscillators. An attempt to generalize this procedure to a $U_q\big(\widehat{sl}(2|1)\big)$ model can be found in [18]. Because of their relevance to systems of interacting electrons in one spatial dimension, vertex models built from representations of $gl(2|1)$ or their q-deformations have been studied using Bethe ansatz in [19].

As the non-simple structure of $sl(n|n)$ or $gl(n|n)$ gives rise to several particular features, the case $m = n$ is excluded in many studies of models related to $sl(m|n)$.

In this paper, the graded Hopf algebra structure of $U_q\big(\widehat{gl}(2|2)\big)$ is studied. $U_q\big(\widehat{sl}(2|2)\big)$ does not admit a quasitriangular structure with respect to the standard definition of coproduct and antipode in terms of its Chevalley generators. A formulation of a quantum affine superalgebra by Drinfeld generators [20, 21] adapted to the particular set of positive simple roots imposed by the intended physical applications [22] permits its extension to $U_q\big(\widehat{gl}(2|2)\big)$. To define the graded Hopf algebra structure, coproduct and antipode need to be introduced for the additional generators. A suitable definition is supplied by the construction of the spectral-dependent universal R-matrix $\mathcal{R}(z)$ of $U_q\big(\widehat{gl}(2|2)\big)$. An explicit expression is given for $\mathcal{R}(z)$ in terms of Drinfeld's generators. Formulae for the coproduct and antipode of the additional generators follow from a partial application of the universal R-matrix to a four-dimensional vector representation associated with the chosen set of simple roots. The implications of the non-simplicity relevant to the construction are discussed.

In view of the applications suggested in [7, 23], an infinite-dimensional module V of $U_q\big(gl(2|2)\big)$ generated from an element characterized by one of the fundamental weights is considered. The model relevant to the description of an integer quantum Hall transition involves an R-matrix acting on the tensor product of the module V with its dual module V^*. In analogy with the case of a quantum affine algebra [17] the quasitriangular structure implies a crossing symmetry which relates this R-matrix to the R-matrix acting on a pair of modules V and thereby facilitates its computation considerably.

Generalization of the investigations to $U_q\big(\widehat{gl}(2n|2n)\big)$ is straightforward. A detailed description of the disordered systems related to $U_q\big(\widehat{gl}(2|2)\big)$-vertex models will be published separately.

Section 2 reviews the definition of the graded Hopf algebra $U_q\big(gl(2|2)\big)$. In section 3 the quantum affine superalgebra $U_q\big(\widehat{sl}(2|2)\big)$ and its Hopf algebra structure are introduced in terms of Chevalley generators. Then using Drinfeld's basis $U_q\big(\widehat{gl}(2|2)\big)$ is defined. Section 4 provides the universal R-matrix. Formulae for the coproduct and antipode completing the Hopf algebra structure of $U_q\big(\widehat{sl}(2|2)\big)$ are derived in section 5. Section 6 deals with the infinite-dimensional $U_q\big(gl(2|2)\big)$ modules.

2. The quantum affine superalgebra $U_q\big(\widehat{gl}(2|2)\big)$

2.1. The Hopf algebra $U_q\big(gl(2|2)\big)$

The quantum deformation $U_q\big(gl(2|2)\big)$ of the universal enveloping superalgebra $U\big(gl(2|2)\big)$

is defined as the associative $\mathbb{Z}_2$-graded algebra over the ring of formal power series $\mathbb{C}[[q-1]]$ generated by $\{e_i, f_i, h_j\}$ with $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ subject to the relations

$$\begin{aligned}
&[h_j, h_{j'}] = 0 \\
&q^{h_j} e_i q^{-h_j} = q^{\overline{a}_{ij}} e_i \\
&q^{h_j} f_i q^{-h_j} = q^{-\overline{a}_{ij}} f_i \\
&[e_i, f_{i'}] = \delta_{i,i'} \frac{q^{h_i} - q^{-h_i}}{q - q^{-1}}
\end{aligned} \tag{1}$$

and

$$\begin{aligned}
&[e_1, e_3] = [f_1, f_3] = 0 \\
&\big[[e_2, e_1]_q, [e_2, e_3]_q\big] = \big[[f_2, f_1]_{q^{-1}}, [f_2, f_3]_{q^{-1}}\big] = 0
\end{aligned} \tag{2}$$

where [,] denotes the Lie superbracket $[x, y] \equiv xy - (-1)^{|x|\cdot|y|} yx$. All simple roots being chosen odd, the Z_2-grading $|\cdot| : U_q\big(gl(2|2)\big) \to Z_2$ assigns the value 1 to each generator e_i, f_i and the value 0 to $1, h_j$. The deformed supercommutators in (2) are defined by

$$[e_i, e_{i'}]_q \equiv e_i e_{i'} + q^{\overline{a}_{ii'}} e_{i'} e_i \qquad [f_i, f_{i'}]_{q^{-1}} \equiv f_i f_{i'} + q^{-\overline{a}_{ii'}} f_{i'} f_i. \tag{3}$$

In (1) $\overline{a}_{jj'} = (\overline{\alpha}_j, \overline{\alpha}_{j'})$ denotes the symmetric bilinear form on the system of positive simple roots $\overline{\alpha}_j$:

$$\overline{a} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}. \tag{4}$$

In terms of the basis $\{\tau_1, \tau_2, \tau_3, \tau_4\}$ with the bilinear form $(\tau_j, \tau_{j'}) = -(-1)^j \delta_{j,j'}$ the simple roots $\overline{\alpha}_j$ and weights Λ_j read $\overline{\alpha}_j = -(-1)^j(\tau_j + \tau_{j+1})$, $j = 1, 2, 3$, $\overline{\alpha}_4 = \tau_1 - \tau_4$ and $\Lambda_j = \sum_{j'=1}^{j} \tau_{j'} - \frac{1}{2}\sum_{j'=1}^{4} \tau_{j'}$. Over $\mathbb{C}[[q-1]]$ one can define the element

$$h = \log q = -\sum_{n=1}^{\infty} \frac{(-1)^n}{n} (q-1)^n. \tag{5}$$

In what follows the notation $q^{x\otimes y} = \exp\big(h\, x \otimes y\big) = \sum_{n=0}^{\infty} \frac{h^n}{n!} (x \otimes y)^n$ will also be used.

$U_q\big(gl(2|2)\big)$ can be endowed with a graded Hopf algebra structure introducing the coproduct

$$\Delta(e_i) = q^{h_i} \otimes e_i + e_i \otimes 1 \qquad \Delta(f_i) = f_i \otimes q^{-h_i} + 1 \otimes f_i \qquad \Delta(q^{h_j}) = q^{h_j} \otimes q^{h_j} \tag{6}$$

the antipode

$$S(e_i) = -q^{-h_i} e_i \qquad S(f_i) = -f_i q^{h_i} \qquad S(q^{h_j}) = q^{-h_j} \tag{7}$$

and the counit

$$\epsilon(e_i) = \epsilon(f_i) = \epsilon(h_j) = 0 \qquad \epsilon(1) = 1. \tag{8}$$

The coproduct satisfies $\Delta(xy) = \Delta(x)\Delta(y)$ with the product operation of the graded Hopf algebra being defined by $(v \otimes w)(x \otimes y) = (-1)^{|w|\cdot|x|} vx \otimes wy$. The antipode S is a $\mathbb{Z}_2$-graded algebra antiautomorphism $S(xy) = (-1)^{|x|\cdot|y|} S(y)S(x)$. $U_q\big(gl(2|2)\big)$ is a quasitriangular graded Hopf algebra with the universal R-matrix $R \in U_q\big(gl(2|2)\big)\hat{\otimes} U_q\big(gl(2|2)\big)$ given by

$$\begin{aligned}
R &= r\, q^{-\frac{1}{2}\sum_{j,j'=1}^{4} \overline{a}_{jj'}(h_j \otimes h_{j'}) - \frac{1}{4}\lambda_0 (h_1+h_3)\otimes(h_1+h_3)} \\
r &= r_1\, r_{1,2}\, r_{1,2,3}\, r_2\, r_{2,3}\, r_3 \\
r_i &= \exp\big((q-q^{-1})\, e_i \otimes f_i\big) \\
r_{1,2} &= \exp_{q^2}\big((q-q^{-1})\,[e_2, e_1]_q \otimes [f_1, f_2]_{q^{-1}}\big) \\
r_{2,3} &= \exp_{q^{-2}}\big(-(q-q^{-1})\,[e_3, e_2]_q \otimes [f_2, f_3]_{q^{-1}}\big) \\
r_{1,2,3} &= \exp\big(-(q-q^{-1})\big(e_3[e_2, e_1]_q - q^{-1}[e_2, e_1]_q e_3\big) \otimes \big([f_1, f_2]_{q^{-1}} f_3 - q f_3 [f_1, f_2]_{q^{-1}}\big)\big)
\end{aligned} \tag{9}$$

where the q-exponent is defined by

$$\exp_p x \equiv \frac{\sum_{n=0}^{\infty} x^n}{(n)_p!}$$

with

$$(n)_p! = (1)_p (2)_p \cdots (n)_p \qquad (l)_p = \frac{1-p^l}{1-p} \qquad p = q^{\pm 2}$$

and $\hat{\otimes}$ denotes a tensor product completed over $\mathbb{C}[[q-1]]$. Because of the non-simple structure of $U_q\big(gl(2|2)\big)$, the parameter λ_0 is not determined.

Use of the algebraic properties of q-exponentials following [24] allows us to verify the quasitriangular structure of $U_q\big(gl(2|2)\big)$:

$$\begin{aligned}
&\Delta'(x) R = R \Delta(x) \qquad \forall\, x \in U_q\big(gl(2|2)\big) \\
&\big(\Delta \otimes \mathrm{id}\big)\, R = R_{13} R_{23} \\
&\big(\mathrm{id} \otimes \Delta\big)\, R = R_{13} R_{12}
\end{aligned} \tag{10}$$

with $\Delta' = \sigma \circ \Delta$, $\sigma(x \otimes y) = (-1)^{|x|\cdot|y|} y \otimes x$ and $R_{12} = R \otimes 1$, $R_{23} = 1 \otimes R$, $R_{13} = (\sigma \otimes \mathrm{id}) R_{23}$. Due to (10) R obeys the universal Yang–Baxter equation

$$R_{12} R_{13} R_{23} = R_{23} R_{13} R_{12}. \tag{11}$$

A pair of representations π_{W_1}, π_{W_2} of $U_q\big(gl(2|2)\big)$ on the modules W_1, W_2 yields a representation of R on $W_1 \otimes W_2$. Given any three $U_q\big(gl(2|2)\big)$ modules W_1, W_2, W_3, a solution of the Yang–Baxter equation

$$R_{12}^{W_1 W_2} R_{13}^{W_1 W_3} R_{23}^{W_2 W_3} = R_{23}^{W_2 W_3} R_{13}^{W_1 W_3} R_{12}^{W_1 W_2} \tag{12}$$

is provided by $R^{W_i W_j} = (\pi_{W_i} \otimes \pi_{W_j}) R \in \mathrm{End}(W_i \otimes W_j)$. $R^{W_i W_j}$ depends on λ_0 only through a constant $C^{ij}(\lambda_0)$. Excluding the generator h_4 from $U_q\big(gl(2|2)\big)$ yields the closed algebra $U_q\big(sl(2|2)\big)$ which is not equipped with a universal R-matrix. The necessity of extending $U_q\big(sl(n|n)\big)$ to $U_q\big(gl(n|n)\big)$ in order to construct particular solutions of the corresponding Yang–Baxter equation has already been stressed in [13]. An element w_k of a given weight representation π_W on the module W is characterized by its weight $\Lambda = \sum_{j=1}^{4} m_j^{(k)} \Lambda_j$ where $\pi(h_j) w_k = m_j^{(k)} w_k$. Because of the non-simple structure of $gl(2|2)$, the component $m_4^{(k)}$ can be varied continuously by an amount independent of k throughout the module without affecting the action of the generators e_i, f_i and h_i, $i = 1, 2, 3$. Then the R-matrix $R_{WW'}$ changes only by a constant $\mathbb{C}$-valued factor as inspection of (9) confirms.

2.2. The quantum affine superalgebra $U_q\big(\widehat{gl}(2|2)\big)$

Introducing an affine root δ with $(\delta,\delta) = (\delta,\tau_i) = 0$ the quantum affine superalgebra $U_q'\big(\widehat{sl}(2|2)\big)$ can be defined based on the set of simple roots $\alpha_0 = \delta - \overline{\alpha}_1 - \overline{\alpha}_2 - \overline{\alpha}_3$, $\alpha_i = \overline{\alpha}_i$, $i = 1, 2, 3$. The set of Chevalley generators $\{e_i, f_i, h_i\}$, $i = 1, 2, 3$ is enlarged by e_0, f_0, h_0 with the $\mathbb{Z}_2$-grading $|e_0| = |f_0| = 1$, $|h_0| = 0$. $U_q'\big(\widehat{sl}(2|2)\big)$ is then defined through the relations

$$
\begin{aligned}
&[h_i, h_k] = 0 \qquad q^{h_i} e_k q^{-h_i} = q^{\hat{a}_{ik}} e_k \qquad q^{h_i} f_k q^{-h_i} = q^{-\hat{a}_{ik}} f_k \\
&[e_i, e_k] = \delta_{i,k} \frac{q^{h_i} - q^{-h_i}}{q - q^{-1}}
\end{aligned}
\tag{13}
$$

and

$$
[e_1, e_3] = [e_0, e_2] = [f_1, f_3] = [f_0, f_2] = 0 \tag{14}
$$

$$
\begin{aligned}
&\big[[e_0, e_1]_q, [e_0, e_3]_q\big] = 0 \qquad \big[[f_0, f_1]_{q^{-1}}, [f_0, f_3]_{q^{-1}}\big] = 0 \\
&\big[[e_1, e_2]_q, [e_1, e_0]_q\big] = 0 \qquad \big[[f_1, f_2]_{q^{-1}}, [f_1, f_0]_{q^{-1}}\big] = 0 \\
&\big[[e_2, e_1]_q, [e_2, e_3]_q\big] = 0 \qquad \big[[f_2, f_1]_{q^{-1}}, [f_2, f_3]_{q^{-1}}\big] = 0 \\
&\big[[e_3, e_2]_q, [e_3, e_0]_q\big] = 0 \qquad \big[[f_3, f_2]_{q^{-1}}, [f_3, f_0]_{q^{-1}}\big] = 0.
\end{aligned}
\tag{15}
$$

Here the deformed supercommutators are defined as in (3) after replacing $\overline{a}$ by

$$
\hat{a} = \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}. \tag{16}
$$

A coproduct and an antipode can be introduced using the expressions given in (6) and (7) for $i, j = 0, 1, 2, 3$. As in the non-affine case, $U_q'\big(\widehat{sl}(2|2)\big)$ has to be extended to $U_q'\big(\widehat{gl}(2|2)\big)$ to obtain a quasitriangular structure. This is achieved by formulating the algebra in terms of the Drinfeld basis [20]. Introduction of a grading operator yields the full quantum affine superalgebra $U_q\big(\widehat{gl}(2|2)\big)$. It is defined as the unital associative $\mathbb{Z}_2$-graded algebra over $\mathbb{C}[[q-1]]$ generated by the Drinfeld generators $\{E_n^{i,\pm}, H_n^j, c\}$ with $n \in \mathbb{Z}$, $i = 1, 2, 3$ and $j = 1, 2, 3, 4$ and the grading operator d subject to the relations

$$
\begin{aligned}
&[c, x] = 0 \qquad \forall\, x \in U_q\big(\widehat{gl}(2|2)\big) \\
&\big[H_n^j, H_{n'}^{j'}\big] = \delta_{n+n',0} \frac{q^{na_{jj'}} - q^{-na_{jj'}}}{n(q - q^{-1})} \frac{\gamma^n - \gamma^{-n}}{q - q^{-1}} \\
&q^{H_0^j} E_n^{i,\pm} q^{-H_0^j} = q^{\pm a_{ij}} E_n^{i,\pm} \\
&\big[H_n^j, E_{n'}^{i,\pm}\big] = \pm \frac{q^{na_{ij}} - q^{-na_{ij}}}{n(q - q^{-1})} E_{n+n'}^{i,\pm} \gamma^{\mp\frac{n}{2}} \\
&\big[E_n^{i,+}, E_{n'}^{i',-}\big] = \delta_{i,i'} \frac{1}{q - q^{-1}} \Big(\gamma^{\frac{1}{2}(n-n')} \Psi_{n+n'}^{i,+} - \gamma^{-\frac{1}{2}(n-n')} \Psi_{n+n'}^{i,-}\Big)
\end{aligned}
\tag{17}
$$

with $\gamma = q^c$ and

$$\Psi^{j,+}(z) \equiv \sum_{n\geqslant 0} \Psi^{j,+}_n z^{-n} = q^{H^j_0} \exp\Big((q-q^{-1}) \sum_{n>0} H^j_n z^{-n}\Big)$$
$$\Psi^{j,-}(z) \equiv \sum_{n\geqslant 0} \Psi^{j,-}_{-n} z^{n} = q^{-H^j_0} \exp\Big(-(q-q^{-1}) \sum_{n>0} H^j_{-n} z^{n}\Big) \tag{18}$$

and

$$\big[E^{i,\pm}_n, E^{i',\pm}_{n'}\big] = 0 \qquad \text{for } a_{ii'} = 0$$
$$E^{i,\pm}_{n+1}E^{i',\pm}_{n'} + q^{\pm a_{ii'}} E^{i',\pm}_{n'} E^{i,\pm}_{n+1} - E^{i',\pm}_{n'+1} E^{i,\pm}_n - q^{\pm a_{ii'}} E^{i,\pm}_n E^{i',\pm}_{n'+1} = 0 \qquad \text{for } a_{ii'} \neq 0 \tag{19}$$
$$\Big[\big[E^{2,\pm}_n, E^{1,\pm}_{n'}\big]_{q^{\pm1}}, \big[E^{2,\pm}_k, E^{3,\pm}_{k'}\big]_{q^{\pm1}}\Big] + \Big[\big[E^{2,\pm}_k, E^{1,\pm}_{n'}\big]_{q^{\pm1}}, \big[E^{2,\pm}_n, E^{3,\pm}_{k'}\big]_{q^{\pm1}}\Big] = 0.$$

The grading operator d is defined by the commutators

$$\big[d, E^{i,\pm}_n\big] = nE^{i,\pm}_n \qquad \big[d, H^j_n\big] = nH^j_n \qquad \big[d,\gamma\big] = \big[d,d\big] = 0. \tag{20}$$

The $\mathbb{Z}_2$-grading $|\cdot| : U_q\big(\widehat{gl}(2|2)\big) \to \mathbb{Z}_2$ assigns 1 to $E^{i,\pm}_n$ and 0 to all remaining generators. In (17)-(19) the symmetric bilinear form $(\alpha_j, \alpha_{j'}) = a_{jj'}$ with $j, j' = 0, 1, 2, 3, 4$ is given by

$$a = \begin{pmatrix} 0 & -1 & 0 & 1 & -2 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 1 \\ -2 & 1 & 0 & 1 & 0 \end{pmatrix}. \tag{21}$$

The deformed supercommutators are defined analogously to (3):

$$[E^{i,\pm}_n, E^{i',\pm}_{n'}]_{\overline{q}} = E^{i,\pm}_n E^{i',\pm}_{n'} + \overline{q}^{\,a_{ii'}} E^{i',\pm}_{n'} E^{i,\pm}_n \qquad \overline{q} = q^{\pm1}. \tag{22}$$

The Chevalley generators of $U_q\big(\widehat{sl}(2|2)\big)$ are related to the Drinfeld generators by the formulae

$$e_i = E^{i,+}_0 \quad f_i = E^{i,-}_0 \quad h_i = H^i_0 \qquad \text{for } i = 1,2,3$$
$$q^{h_0} = \gamma q^{-h_1-h_2-h_3}$$
$$e_0 = \Big[E^{3,-}_0, \big[E^{2,-}_0, E^{1,-}_1\big]_q\Big]_q q^{-h_1-h_2-h_3} \tag{23}$$
$$f_0 = q^{h_1+h_2+h_3}\Big[\big[E^{1,+}_{-1}, E^{2,+}_0\big]_{q^{-1}}, E^{3,+}_0\Big]_{q^{-1}}$$

where

$$\Big[E^{i,\pm}_m, \big[E^{k,\pm}_n, E^{k',\pm}_{n'}\big]_{q'}\Big]_{\overline{q}} = -\overline{q}^{\,a_{ik}+a_{ik'}}\Big[\big[E^{k,\pm}_n, E^{k',\pm}_{n'}\big]_{q'}, E^{i,\pm}_m\Big]_{\overline{q}^{-1}}$$
$$= E^{i,\pm}_m\big[E^{k,\pm}_n, E^{k',\pm}_{n'}\big]_{q'} - \overline{q}^{\,a_{ik}+a_{ik'}}\big[E^{k,\pm}_n, E^{k',\pm}_{n'}\big]_{q'} E^{i,\pm}_m \qquad q', \overline{q} = \pm 1. \tag{24}$$

This isomorphism allows us to evaluate coproduct and antipode of the Drinfeld generators $\{E^{i,\pm}_n, H^i_n,\ i = 1,2,3\}$ from their definitions in terms of Chevalley generators. The coproducts of c and d are $\Delta(c) = c\otimes 1 + 1\otimes c$ and $\Delta(d) = d\otimes 1 + 1\otimes d$. In order to establish the standard graded Hopf algebra structure of $U_q\big(\widehat{gl}(2|2)\big)$ the coproduct and antipode of H^4_n still need to be introduced suitably. Their definitions are implied by the construction of the spectral dependent universal R-matrix for the quantum affine super algebra. The antipode relates the Drinfeld generators of $U_q(\widehat{sl}(2|2))$ to a second set of generators by

$$\begin{aligned}\hat{H}_n^i &= -\gamma^n S(H_n^i) \qquad S(\hat{H}_n^i) = -\gamma^{-n} H_n^i \\ \hat{E}_n^{i,\pm} &= -\gamma^n q^{\pm h_i} S(E_n^{i,\pm}) \qquad S(\hat{E}_n^{i,\pm}) = -\gamma^{-n} q^{\mp h_i} E_n^{i,\pm}.\end{aligned} \tag{25}$$

The defining relations of the set $\{\hat{E}_n^{i,\pm}, \hat{H}_n^i, c, d\}$ with $m \in \mathbb{Z}, n \in \mathbb{Z} - \{0\}$ as well as its relation to the Chevalley generators are obtained from (17)–(19) and (23) by the replacement $q \to q^{-1}$. Each set is extended to a realization of $U_q(\widehat{gl}(2|2))$ introducing the generators H_n^4 and $\hat{H}_n^4$, respectively. The antipodes of the latter are defined as

$$S(H_n^4) = -\gamma^{-n} \hat{H}_n^4 \qquad S(\hat{H}_n^4) = -\gamma^{-n} H_n^4. \tag{26}$$

A different (graded) Hopf algebra structure originally due to Drinfeld [25, 26] shows a very simple coproduct in terms of the Drinfeld generators. Although not frequently used, this coproduct allows one to derive quantum parafermions for quantum affine algebras [27].

3. The universal R-matrix

To set up the quasitriangular structure for the affine case, define automorphisms D_z of $U_q(\widehat{gl}(2|2)) \otimes \mathbb{C}[z, z^{-1}]$ as

$$D_z(E_n^{i,\pm}) = z^n E_n^{i,\pm} \qquad D_z(H_n^j) = z^n H_n^j \qquad D_z(d) = d \tag{27}$$

and maps

$$\Delta_z(x) = (D_z \otimes \mathrm{id})\Delta(x) \quad \Delta'_z(x) = (D_z \otimes \mathrm{id})\Delta'(x) \qquad \forall\, x \in U_q(\widehat{gl}(2|2)). \tag{28}$$

By means of D_z, the spectral dependent universal R-matrix $\mathcal{R}(z)$ of $U_q(\widehat{gl}(2|2))$ can be introduced [17] as

$$\mathcal{R}(z) = (D_z \otimes \mathrm{id})(\mathcal{R}) \tag{29}$$

with the following properties:

$$\mathcal{R}(z)\, \Delta_z(x) = \Delta'_z(x)\, \mathcal{R}(z) \tag{30}$$

$$\begin{aligned}(\Delta_z \otimes \mathrm{id})(\mathcal{R}(w)) &= \mathcal{R}_{13}(zw)\mathcal{R}_{23}(w) \\ (\mathrm{id} \otimes \Delta_w)(\mathcal{R}(zw)) &= \mathcal{R}_{13}(z)\mathcal{R}_{12}(zw)\end{aligned} \tag{31}$$

where $\mathcal{R}_{12}(z) = \mathcal{R}(z) \otimes 1$, $\mathcal{R}_{23}(z) = 1 \otimes \mathcal{R}(z)$, $\mathcal{R}_{13}(z) = (\sigma \otimes \mathrm{id})(\mathcal{R}_{23}(z))$. Equations (30) and (31) imply the Yang–Baxter equation with a spectral parameter for $\mathcal{R}(z)$:

$$\mathcal{R}_{12}(z)\mathcal{R}_{13}(zw)\mathcal{R}_{23}(w) = \mathcal{R}_{23}(w)\mathcal{R}_{13}(zw)\mathcal{R}_{12}(z). \tag{32}$$

Generally, $\mathcal{R}$ has the form $\mathcal{R} = \sum_i e_i \otimes e^i$ where $\{e_i\}$ ($\{e^i\}$) is a suitable basis of $U_q(\hat{b}_+)$ $(U_q(\hat{b}_-))$. $\hat{b}_+$ and $\hat{b}_-$ are the Borel subalgebras of $\widehat{gl}(2|2)$ and $U_q(\hat{b}_\pm)$ denote the corresponding graded Hopf subalgebras generated by $\{E_0^{i,+},\ E_n^{i,+},\ E_n^{i,-},\ H_0^j,\ H_n^j,\ \ n > 0,\ c,\ d\}$ and $\{E_0^{i,-},\ E_{-n}^{i,-},\ E_{-n}^{i,+},\ H_0^j,\ H_{-n}^j,\ \ n > 0,\ c,\ d\}$ over $\mathbb{C}[[q-1]]$, respectively. Properties (31) are established by means of the quantum double construction [28]. A procedure to obtain explicit formulae for the universal R-matrix of affine quantum algebras in terms of the Cartan–Weyl basis built from the Chevalley generators can be found in [29]. Since a triangular structure of this form does not exist for $U_q(\widehat{sl}(2|2))$ generated by this basis, the Drinfeld generators are used for the construction of the universal R-matrix along similar lines.

Let the normal ordering in the positive root space Δ_+ of $U_q\big(\widehat{sl}(2|2)\big)$ be fixed by $\alpha_1,\dots,\alpha_1+k_1\delta,\dots,\alpha_1+\alpha_2,\dots,\alpha_1+\alpha_2+k_2\delta,\dots,\alpha_1+\alpha_2+\alpha_3,$

$$\dots,\alpha_1+\alpha_2+\alpha_3+k_3\delta,\dots,\alpha_2,\dots,\alpha_2+k_4\delta,\dots,\alpha_2+\alpha_3,$$

$$\dots,\alpha_2+\alpha_3+k_5\delta,\dots,\alpha_3,\dots,\alpha_3+k_6,$$

$$\dots,\delta,2\delta,3\delta,\dots,\alpha_0+\alpha_1+\alpha_2+l_1\delta,$$

$$\dots,\alpha_0+\alpha_1+\alpha_2,\dots,\alpha_0+\alpha_1+\alpha_3+l_2\delta,$$

$$\dots,\alpha_0+\alpha_1+\alpha_3,\dots,\alpha_0+\alpha_1+l_3\delta,\dots,\alpha_0+\alpha_1,$$

$$\dots,\alpha_0+\alpha_2+\alpha_3+l_4\delta,\dots,\alpha_0+\alpha_2+\alpha_3,$$

$$\dots,\alpha_0+\alpha_3+l_5\delta,\dots,\alpha_0+\alpha_3,\dots,\alpha_0+l_6\delta,\dots,\alpha_0.$$

A solution of (30) for $x\in U_q\big(\widehat{sl}(2|2)\big)$ in terms of the Drinfeld generators is given by (29) with

$$\mathcal{R}=\breve{R}\,q^{-\frac{1}{2}\sum_{i,j=1}^{4}a_{ij}(h_i\otimes h_j)-\frac{1}{4}\lambda_0(h_1+h_3)\otimes(h_1+h_3)-c\otimes d-d\otimes c} \tag{33}$$

with

$$\breve{R}=R^{<}\,R^{(\delta)}\,R^{>}$$

$$R^{<}=R_{\alpha_1}R_{\alpha_1+\alpha_2}R_{\alpha_1+\alpha_2+\alpha_3}R_{\alpha_2}R_{\alpha_2+\alpha_3}R_{\alpha_3}$$

$$R_{\alpha_i}=\prod_{n=0}^{\rightarrow}\exp\left((q-q^{-1})\gamma^{-n}E_n^{i,+}\otimes E_{-n}^{i,-}\gamma^n\right)$$

$$R_{\alpha_1+\alpha_2}=\prod_{n=0}^{\rightarrow}\exp_{q^2}\left((q-q^{-1})\gamma^{-n}\big[E_0^{2,+},E_n^{1,+}\big]_q\otimes\big[E_{-n}^{1,-},E_0^{2,-}\big]_{q^{-1}}\gamma^n\right)$$

$$R_{\alpha_2+\alpha_3}=\prod_{n=0}^{\rightarrow}\exp_{q^{-2}}\left(-(q-q^{-1})\gamma^{-n}\big[E_0^{3,+},E_n^{2,+}\big]_q\otimes\big[E_{-n}^{2,-},E_0^{3,-}\big]_{q^{-1}}\gamma^n\right)$$

$$\begin{aligned}R_{\alpha_1+\alpha_2+\alpha_3}=\prod_{n=0}^{\rightarrow}\exp\Big(&-(q-q^{-1})\gamma^{-n}\Big[E_0^{3,+},\big[E_0^{2,+},E_n^{1,+}\big]_q\Big]_q\\&\otimes\Big[\big[E_{-n}^{1,-},E_0^{2,-}\big]_{q^{-1}},E_0^{3,-}\Big]_{q^{-1}}\gamma^n\Big)\end{aligned} \tag{34}$$

$$R^{(\delta)}=\exp\left(-\frac{1}{2}(q-q^{-1})\sum_{n=1}^{\infty}\frac{n}{[n]}\sum_{i,j=1}^{4}a_{ij}\gamma^{-\frac{1}{2}n}H_n^i\otimes H_{-n}^j\gamma^{\frac{1}{2}n}\right) \tag{35}$$

$$R^{>}=R_{\alpha_0+\alpha_1+\alpha_2}R_{\alpha_0+\alpha_1+\alpha_3}R_{\alpha_0+\alpha_1}R_{\alpha_0+\alpha_2+\alpha_3}R_{\alpha_0+\alpha_3}R_{\alpha_0}$$

$$R_{\alpha_0+\alpha_i+\alpha_j}=\prod_{n=1}^{\leftarrow}\exp\left(-(q-q^{-1})q^{-h_k}E_n^{k,-}\otimes E_{-n}^{k,+}q^{h_k}\right)\qquad k\neq 0,i,j$$

$$R_{\alpha_0+\alpha_3}=\prod_{n=1}^{\leftarrow}\exp_{q^2}\left((q-q^{-1})q^{-h_1-h_2}\big[E_n^{1,-},E_0^{2,-}\big]_{q^{-1}}\otimes\big[E_0^{2,+},E_{-n}^{1,+}\big]_q q^{h_1+h_2}\right)$$

$$R_{\alpha_0+\alpha_1} = \prod_{n=1}^{\overleftarrow{\infty}} \exp_{q^{-2}} \left(-(q-q^{-1}) q^{-h_2-h_3} \left[E_n^{2,-}, E_0^{3,-} \right]_{q^{-1}} \otimes \left[E_0^{3,+}, E_{-n}^{2,+} \right]_q q^{h_2+h_3} \right)$$

$$R_{\alpha_0} = \prod_{n=1}^{\overleftarrow{\infty}} \exp \left((q-q^{-1}) q^{-h_1-h_2-h_3} \left[\left[E_n^{1,-}, E_0^{2,-} \right]_{q^{-1}}, E_0^{3,-} \right]_{q^{-1}} \right.$$

$$\left. \otimes \left[E_0^{3,+}, \left[E_0^{2,+}, E_{-n}^{1,+} \right]_q \right]_q q^{h_1+h_2+h_3} \right). \tag{36}$$

In (34) and (36) the direction of increasing n is indicated by the arrow. For any set of parameters $\{\lambda_n, n > 0\}$, the expression

$$\breve{R} \cdot \exp\left(\frac{1}{4}(q-q^{-1}) \sum_{n=1}^{\infty} \frac{n}{[n]} \lambda_n \gamma^{-\frac{1}{2}n} (H_n^1 + H_n^3) \otimes (H_{-n}^1 + H_{-n}^3) \gamma^{\frac{1}{2}n} \right)$$

yields another solution of (30) for $x \in U_q\big(\widehat{sl}(2|2)\big)$. An expression for $\Delta\big(\Psi^{4,\pm}(z)\big)$ independent of the choice of $\{\lambda_n\}$ will be provided in section 4. Imposing the intertwining condition (30) for any $x \in U_q\big(\widehat{gl}(2|2)\big)$ yields $\lambda_n = 0$ for $n > 0$. Since $S^2 = \mathrm{id}$ due to the particular choice of simple roots and (26), its inverse satisfies [30]

$$\mathcal{R}^{-1}(z) = \big(S \otimes \mathrm{id}\big)\big(\mathcal{R}(z)\big) = \big(\mathrm{id} \otimes S\big)\big(\mathcal{R}(z)\big). \tag{37}$$

The dependence of the universal R-matrix of $U_q\big(\widehat{gl}(2|2)\big)$ on d is contained entirely in the second term of the right-hand side of (33). Thus, following [17] the universal R-matrix for $U_q'\big(\widehat{gl}(2|2)\big)$ is defined by

$$R(z) = \mathcal{R}(z)\, q^{(c\otimes d + d\otimes c)}. \tag{38}$$

Writing $R_{ij}(z) = \sum_{n=0}^{\infty} R_{ij,n} z^n$ one may introduce the notation

$$\begin{aligned} R_{12}(zq^{\pm c_3}) &= \sum_{n=0}^{\infty} R_{12,n}\, z^n \big(\mathrm{id} \otimes \mathrm{id} \otimes q^{\pm nc}\big) \\ R_{13}(zq^{\pm c_2}) &= \sum_{n=0}^{\infty} R_{13,n}\, z^n \big(\mathrm{id} \otimes q^{\pm nc} \otimes \mathrm{id}\big) \\ R_{23}(zq^{\pm c_1}) &= \sum_{n=0}^{\infty} R_{23,n}\, z^n \big(q^{\pm nc} \otimes \mathrm{id} \otimes \mathrm{id}\big). \end{aligned} \tag{39}$$

Then for $R^-(z) = R(z)$, $R^+(z) = \sigma\big(R^{-1}(z)\big)$ the Yang–Baxter equation reads

$$\begin{aligned} R_{12}^{\pm}(z) R_{13}^{\pm}(zwq^{\mp c_2}) R_{23}^{\pm}(w) &= R_{23}^{\pm}(w) R_{13}^{\pm}(zwq^{\pm c_2}) R_{12}^{\pm}(z) \\ R_{12}^{-}(zq^{-c_3}) R_{13}^{-}(zw) R_{23}^{+}(w) &= R_{23}^{+}(w) R_{13}^{-}(zw) R_{12}^{-}(zq^{c_3}). \end{aligned} \tag{40}$$

From (31) and (37) one obtains the comultiplication and antipode formulae

$$\begin{aligned} &\big(\Delta \otimes \mathrm{id}\big)\, R^-(z) = R_{13}^-(z) R_{23}^-\big(zq^{c_1}\big) && \big(S \otimes \mathrm{id}\big)\, R^-(z) = \big(R^-(zq^{-c_1})\big)^{-1} \\ &\big(\mathrm{id} \otimes \Delta\big)\, R^-(z) = R_{13}^-(z) R_{12}^-\big(zq^{-c_3}\big) && \big(\mathrm{id} \otimes S\big)\, R^-(z) = \big(R^-(zq^{c_2})\big)^{-1} \\ &\big(\Delta \otimes \mathrm{id}\big)\, R^+(z) = R_{13}^+\big(zq^{-c_2}\big) R_{23}^+(z) && \big(S \otimes \mathrm{id}\big)\, R^+(z) = \big(R^+(zq^{c_1})\big)^{-1} \\ &\big(\mathrm{id} \otimes \Delta\big)\, R^+(z) = R_{13}^+\big(zq^{c_2}\big) R_{12}^+(z) && \big(\mathrm{id} \otimes S\big)\, R^+(z) = \big(R^+(zq^{-c_2})\big)^{-1}. \end{aligned} \tag{41}$$

4. Vector representation and RSTS generators

For any zero-level representation $\pi_W : U'_q\big(\widehat{gl}(2|2)\big) \to \mathrm{End}(W)$ the associated L-operators are introduced according to [33, 17, 34]:

$$L^{\pm,W}(z) = \big(\pi_W \otimes \mathrm{id}\big)\, R^{\pm}(z). \tag{42}$$

Taking the image of (40) in $W_z \otimes W'_w$ one obtains the RLL relations

$$\begin{aligned} R_{12}^{\pm,WW'}\left(\frac{z}{w}\right) L_1^{\pm,W}(z) L_2^{\pm,W'}(w) &= L_2^{\pm,W'}(w) L_1^{\pm,W}(z) R_{12}^{\pm,WW'}\left(\frac{z}{w}\right) \\ R_{12}^{-,WW'}\left(q^{-c}\frac{z}{w}\right) L_1^{-,W}(z) L_2^{+,W'}(w) &= L_2^{+,W'}(w) L_1^{-,W}(z) R_{12}^{-,WW'}\left(q^{c}\frac{z}{w}\right) \end{aligned} \tag{43}$$

where

$$R^{\pm,WW'}(z) = \big(\pi_W \otimes \pi_{W'}\big)\, R^{\pm}(z). \tag{44}$$

Choosing a basis $\{w_i\}$ in W, one can regard $L^{\pm,W}(z)$ as matrices with matrix elements $L_{ij}^{\pm,W}(z) \in U_q\big(\widehat{gl}(2|2)\big)$. Equation (41) yields the comultiplication formulae for the L-operators:

$$\begin{aligned} \Delta L_{ij}^{-,W}(z) &= \sum_{l=1}^{4} (-1)^{(|i|+|l|)\cdot(|j|+|l|)} L_{lj}^{-,W}\big(zq^{-c_2}\big) \otimes L_{il}^{-,W}(z) \\ \Delta L_{ij}^{+,W}(z) &= \sum_{l=1}^{4} (-1)^{(|i|+|l|)\cdot(|j|+|l|)} L_{lj}^{+,W}(z) \otimes L_{il}^{+,W}\big(zq^{c_1}\big). \end{aligned} \tag{45}$$

A finite-dimensional module of $U_q\big(sl(2|2)\big)$ is provided by $W_{(4)} = \mathbb{C}w_1 \oplus \mathbb{C}w_2 \oplus \mathbb{C}w_3 \oplus \mathbb{C}w_4$ with the action

$$\begin{aligned} h_i w_k &= \big(\delta_{k,i} + \delta_{k,i+1}\big)\,(-1)^i\, w_k \qquad i = 1, 2, 3 \\ e_i w_k &= \delta_{k,i}\,(-1)^i\, w_{i+1} \\ f_i w_k &= \delta_{k,i+1} w_i \end{aligned} \tag{46}$$

and the Z_2-grading $|w_1| = |w_3| = 0$, $|w_2| = |w_4| = 1$. Making use of the evaluation homomorphism ρ [31, 32] allows us to equip $W^{(4)}$ with a representation $\pi_{W^{(4)}} : U'_q\big(\widehat{sl}(2|2)\big) \to \mathrm{End}(W^{(4)})$ completing (46) [22] by

$$\begin{aligned} h_0 w_k &= -(h_1 + h_2 + h_3) w_k \\ e_0 w_k &= -q\delta_{k,4} w_1 \\ f_0 w_k &= -q^{-1}\delta_{k,1} w_4. \end{aligned} \tag{47}$$

In terms of the Drinfeld generators the action of $U'_q\big(\widehat{sl}(2|2)\big)$ is obtained from (46) and (47) by means of (23):

$$\begin{aligned} E_n^{i,+} w_k &= \delta_{k,i}\,(-1)^i\, q^{(1+(-1)^i)\frac{1}{2}n} w_{i+1} \qquad i = 1, 2, 3 \\ E_n^{i,-} w_k &= \delta_{k,i+1} q^{(1+(-1)^i)\frac{1}{2}n} w_i \\ H_n^i w_k &= \big(\delta_{k,i} + \delta_{k,i+1}\big)\,(-1)^i\, q^{(1-(-1)^i)\frac{1}{2}n} \frac{[n]}{n} w_k. \end{aligned} \tag{48}$$

Since the universal R-matrix (33)–(36) only contains H_n^4 in the combinations $H_n^4 \otimes (H_{-n}^1 + H_{-n}^3)$ and $(H_n^1 + H_n^3) \otimes H_{-n}^4$ and $(H_{\pm n}^1 + H_{\pm n}^3) w_k = -q^{\pm\frac{1}{2}n} w_k\ \forall k$, the R-matrix

$R^{W^{(4)}W^{(4)}}(z) \equiv R^{-,W^{(4)}W^{(4)}}(z)$ is determined by (48) up to a function of z constant throughout $W^{(4)} \otimes W^{(4)}$. Its non-vanishing entries read

$$\begin{aligned}
&R^{33}_{33}(z) = R^{11}_{11}(z) \\
&R^{44}_{44}(z) = R^{22}_{22}(z) = \frac{q^2 - z}{1 - q^2 z} R^{11}_{11}(z) \\
&R^{ij}_{ij}(z) = \frac{(1-z)q}{1-q^2 z} R^{11}_{11}(z) \\
&R^{ji}_{ij}(z) = (-1)^{|i|\cdot|j|} \frac{1-q^2}{1-q^2 z} R^{11}_{11}(z) \qquad \text{for } i < j \\
&R^{ji}_{ij}(z) = (-1)^{|i|\cdot|j|} \frac{(1-q^2)z}{1-q^2 z} R^{11}_{11}(z) \qquad \text{for } i > j.
\end{aligned} \tag{49}$$

To evaluate the L-operators (42) corresponding to $W^{(4)}$, it is convenient to write the action of the generators H^4_n as

$$\begin{aligned}
&H^4_0 w_k = -\big(\delta_{k,1} - \delta_{k,4} - \kappa_0\big) w_k \\
&H^4_n w_k = -\big(\delta_{k,1} - \delta_{k,4} - \kappa_n\big) \frac{[n]}{n} w_k \qquad n \neq 0.
\end{aligned} \tag{50}$$

$L^{\pm,W^{(4)}}$ depends on parameters $\{\kappa_0, \kappa_n\}$ and $\{\lambda_0, \lambda_n\}$ only through the common factor

$$f^{\pm}(z) = \exp\left(\pm\frac{1}{2}(q - q^{-1}) \sum_{n=1}^{\infty} (\kappa_{\mp n} - \lambda_{\mp n}) \left(H^1_{\pm n} + H^3_{\pm n}\right) \gamma^{\mp\frac{n}{2}} z^n\right) q^{\pm\frac{1}{2}(\kappa_0 - \lambda_0)(h_1 + h_3)}. \tag{51}$$

The dependence of the remaining part on the Drinfeld generators is conveniently formulated by means of a triangular decomposition:

$$L^{\pm,W^{(4)}} = f^{\pm} \cdot \begin{pmatrix} 1 & & \cdots & 0 \\ A^{\pm}_{21} & 1 & & \vdots \\ A^{\pm}_{31} & A^{\pm}_{32} & 1 & \\ A^{\pm}_{41} & A^{\pm}_{42} & A^{\pm}_{43} & 1 \end{pmatrix} \begin{pmatrix} k^{\pm}_1 & & \cdots & 0 \\ & k^{\pm}_2 & & \vdots \\ \vdots & & k^{\pm}_3 & \\ 0 & \cdots & & k^{\pm}_4 \end{pmatrix} \begin{pmatrix} 1 & B^{\pm}_{12} & B^{\pm}_{13} & B^{\pm}_{14} \\ & 1 & B^{\pm}_{23} & B^{\pm}_{24} \\ \vdots & & 1 & B^{\pm}_{34} \\ 0 & \cdots & & 1 \end{pmatrix} \tag{52}$$

where the argument z is omitted for simplicity. The Drinfeld generators are related to the entries of (52) by

$$\begin{aligned}
&\Psi^{i,\pm}(z^{\mp 1}) = \left(k^{\pm}_i\big(zq^{\pm\frac{1}{2}(1+(-1)^i)}\gamma^{\pm\frac{1}{2}}\big)\right)^{-1} k^{\pm}_{i+1}\big(zq^{\pm\frac{1}{2}(1+(-1)^i)}\gamma^{\pm\frac{1}{2}}\big) \qquad \text{for } i = 1, 2, 3 \\
&\Psi^{4,\pm}(z^{\mp 1}) = \left(k^{\pm}_1\big(zq^{\pm 1}\gamma^{\pm\frac{1}{2}}\big) k^{\pm}_4\big(zq^{\pm 1}\gamma^{\pm\frac{1}{2}}\big)\right)^{-1}
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
&(q - q^{-1}) \sum_{n=\frac{1}{2}(1\mp 1)}^{\infty} E^{i,+}_{\pm n} z^n = \pm(-1)^i B^{\pm}_{i\,i+1}\big(zq^{\pm\frac{1}{2}(1+(-1)^i)}\gamma^{\frac{1}{2}(1\pm 1)}\big) \\
&(q - q^{-1}) \sum_{n=\frac{1}{2}(1\pm 1)}^{\infty} E^{i,-}_{\pm n} z^n = \pm A^{\pm}_{i+1\,i}\big(zq^{\pm\frac{1}{2}(1+(-1)^i)}\gamma^{-\frac{1}{2}(1\mp 1)}\big) \qquad \text{for } i = 1, 2, 3.
\end{aligned} \tag{54}$$

The choice $\kappa_n - \lambda_n = 0\ \forall\, n$ corresponds to the generalization of the Reshetikhin–Semenov-Tian-Shansky basis for the quantum affine algebra $U_q\big(\widehat{sl}(N)\big)$ [25] to $U_q\big(\widehat{gl}(N|N)\big)$ in the case $N = 2$. With $\kappa_0 - \lambda_0 = 1$, $\kappa_n - \lambda_n = q^n$, $\kappa_{-n} - \lambda_{-n} = 1$ for $n > 0$ the coproduct $\Delta(H^4_{\pm n})$ can be expressed via the coproduct of

$$L^{\pm}_{11}(\tilde{z}^{\pm}) = q^{\mp\frac{1}{2}(h_2+h_4)}\ \exp\left(\mp\frac{1}{2}(q-q^{-1})\sum_{n=1}^{\infty}\big(H^2_{\pm n}+H^4_{\pm n}\big)z^n\right)\qquad \text{with } \tilde{z}^{\pm} = zq^{\pm 1}\gamma^{\pm\frac{1}{2}} \tag{55}$$

which is obtained from (45) as

$$\begin{aligned}\Delta\left(L^{\pm}_{11}(\tilde{z}^{\pm})\right) = {} & \left(1\otimes L^{\pm}_{11}(\tilde{z}^{\pm}q^{\frac{1}{2}(1\pm 2)c_1})\right)\\ & \cdot\left(1\otimes 1 - \sum_{i=2}^{4}(-1)^i A^{\pm}_{i1}(\tilde{z}^{\pm}q^{-\frac{1}{2}(1\mp 2)c_2})\otimes B^{\pm}_{1i}(\tilde{z}^{\pm}q^{\frac{1}{2}(1\pm 2)c_1})\right)\\ & \cdot\left(L^{\pm}_{11}(\tilde{z}^{\pm}q^{-\frac{1}{2}(1\mp 2)c_2})\otimes 1\right)\end{aligned} \tag{56}$$

where

$$\begin{aligned} A^{\pm}_{31}(z) &= \pm(q-q^{-1})\sum_{n=\frac{1}{2}(1\pm 1)}^{\infty}\big[E^{1,-}_{\pm n}, E^{2,-}_{0}\big]_{q^{-1}}\gamma^{\frac{1}{2}n(1\mp 1)}z^n\\ A^{\pm}_{41}(z) &= \pm(q-q^{-1})\sum_{n=\frac{1}{2}(1\pm 1)}^{\infty}\Big[\big[E^{1,-}_{\pm n}, E^{2,-}_{0}\big]_{q^{-1}}, E^{3,-}_{0}\Big]_{q^{-1}}\gamma^{\frac{1}{2}n(1\mp 1)}z^n\\ B^{\pm}_{13}(z) &= \mp(q-q^{-1})\sum_{n=\frac{1}{2}(1\mp 1)}^{\infty}\big[E^{2,+}_{0}, E^{1,+}_{\pm n}\big]_{q}\gamma^{-\frac{1}{2}n(1\pm 1)}z^n\\ B^{\pm}_{14}(z) &= \pm(q-q^{-1})\sum_{n=\frac{1}{2}(1\mp 1)}^{\infty}\Big[E^{3,+}_{0}, \big[E^{2,+}_{0}, E^{1,+}_{\pm n}\big]_{q}\Big]_{q}\gamma^{-\frac{1}{2}n(1\pm 1)}z^n. \end{aligned} \tag{57}$$

Similar formulae can be derived considering the operators $K^{\pm}(z) = \big(\mathrm{id}\otimes\pi_{W^{(4)}}\big)\,R^{\mp}(z)$ with the coproducts $\Delta\big(K^+_{ij}(z)\big) = \sum_{l=1}^4 K^+_{il}(z)\otimes K^+_{lj}(zq^{c_1})$ and $\Delta\big(K^-_{ij}(z)\big) = \sum_{l=1}^4 K^-_{il}(zq^{-c_2})\otimes K^-_{lj}(z)$ according to (41). Then $\Delta(H^2_{\pm n} - H^4_{\pm n})$ follows from the coproduct of

$$K^{\pm}_{44}(\tilde{z}^{\pm}) = q^{\mp\frac{1}{2}(h_2-h_4)}\ \exp\left(\mp\frac{1}{2}(q-q^{-1})\sum_{n=1}^{\infty}\big(H^2_{\pm n}-H^4_{\pm n}\big)z^n\right) \tag{58}$$

which is given by

$$\begin{aligned}\Delta\left(K^{\pm}_{44}\big(\tilde{z}^{\pm}\big)\right) = {} & \left(K^{\pm}_{44}\big(\tilde{z}^{\pm}q^{-\frac{1}{2}(1\mp 2)c_2}\big)\otimes 1\right)\\ & \cdot\left(1\otimes 1 + \sum_{i=1}^{3}(-1)^i A^{\pm}_{4i}\big(\tilde{z}^{\pm}q^{-\frac{1}{2}(1\mp 2)c_2}\big)\otimes B^{\pm}_{i4}\big(\tilde{z}^{\pm}q^{\frac{1}{2}(1\pm 2)c_1}\big)\right)\\ & \cdot\left(1\otimes K^{\pm}_{44}\big(\tilde{z}^{\pm}q^{\frac{1}{2}(1\pm 2)c_1}\big)\right)\end{aligned} \tag{59}$$

with

$$B_{24}^{\pm}(z) = \mp(q - q^{-1}) \sum_{n=\frac{1}{2}(1\mp1)}^{\infty} \left[E_0^{3,+}, E_{\pm n}^{2,+}\right]_q q^{\mp n} \gamma^{-\frac{1}{2}n(1\pm1)} z^n$$
$$A_{42}^{\pm}(z) = \mp(q - q^{-1}) \sum_{n=\frac{1}{2}(1\pm1)}^{\infty} \left[E_{\pm n}^{2,-}, E_0^{3,-}\right]_{q^{-1}} q^{\mp n} \gamma^{\frac{1}{2}n(1\mp1)} z^n. \tag{60}$$

Combining equations (56) and (59) provides a definition of the coproduct of $\Psi^{4,\pm}(z)$ consistent with the quasitriangular structure. It can easily be seen that a different choice of $\{\kappa_n - \lambda_n\}$ gives rise to the same formulae for the coproduct of the right hand sides of (55) and (58) by observing that the coproduct of $(H_{\pm n}^1 + H_{\pm n}^3)$ is given by

$$\Delta(H_{\pm n}^1 + H_{\pm n}^3) = (H_{\pm n}^1 + H_{\pm n}^3) \otimes \gamma^{-\frac{1}{2}n(1\mp2)} + \gamma^{\frac{1}{2}n(1\pm2)} \otimes (H_{\pm n}^1 + H_{\pm n}^3) \tag{61}$$

independent of $\{\kappa_n, \lambda_n\}$. Thus equation (56) (or equation (59)) can be used to set up the intertwining condition (30) for all $x \in U_q\big(\widehat{gl}(2|2)\big)$. This requirement fixes $\mathcal{R}$ as given in (33)–(36) up to the parameter λ_0.

Finally, the antipode of $\Psi^{4,\pm}(z)$ is obtained making use of (37) to evaluate the antipodes of $L_{11}^{\pm}(z)$ and $K_{44}^{\pm}(z)$:

$$S\big(L_{11}^{\pm}(z)\big) = \big(L_{11}^{\pm}(z\gamma^{\mp1})\big)^{-1} + \big(\Psi^{1,\pm}(z^{\mp1}\gamma^{\mp1})\Psi^{3,\pm}(z^{\mp1}\gamma^{\mp1})\big)^{-\frac{1}{2}} \cdot \sum_{i=2}^{4} B_{1i}^{\pm}(z\gamma^{\mp1})\left(k_i^{\pm}(z\gamma^{\mp1})\right)^{-1} A_{i1}^{\pm}(z\gamma^{\mp1})$$
$$S\big(K_{44}^{\pm}(z)\big) = \big(K_{44}^{\pm}(z\gamma^{\mp1})\big)^{-1} + \big(\Psi^{1,\pm}(z^{\mp1}\gamma^{\mp1})\Psi^{3,\pm}(z^{\mp1}\gamma^{\mp1})\big)^{-\frac{1}{2}} \cdot \sum_{i=1}^{3} A_{4i}^{\pm}(z\gamma^{\mp1})\, k_i^{\pm}(z\gamma^{\mp1})\, B_{i4}^{\pm}(z\gamma^{\mp1}). \tag{62}$$

5. An infinite-dimensional $U_q\big(gl(2|2)\big)$ module

Motivated by physical applications to two-dimensional disordered systems [7] the remainder of this paper focuses on evaluation modules associated with a pair of infinite-dimensional $U_q\big(gl(2|2)\big)$ representations. The modules V and $\hat{V}$ are generated by the action of $U_q\big(gl(2|2)\big)$ on the elements $v_{0,0}$ and $\hat{v}_{0,0}$ characterized by

$$h_1 v_{0,0} = h_3 v_{0,0} = 0 \qquad h_2 v_{0,0} = h_4 v_{0,0} = -v_{0,0} \qquad e_i v_{0,0} = 0$$
$$h_1 \hat{v}_{0,0} = h_3 \hat{v}_{0,0} = 0 \qquad h_2 \hat{v}_{0,0} = h_4 \hat{v}_{0,0} = \hat{v}_{0,0} \qquad f_i \hat{v}_{0,0} = 0 \tag{63}$$

for $i = 1, 2, 3$. The basis vectors of V and $\hat{V}$ are conveniently denoted by $v_{k,p}$ and $\hat{v}_{k,p}$ respectively [22], where $k = 0, 1, 2, 3$ and $p = 0, 1, 2, \ldots$ and the $\mathbb{Z}_2$-grading is defined by $|v_{0,p}| = |\hat{v}_{0,p}| = |v_{3,p}| = |\hat{v}_{3,p}| = 0$, $|v_{1,p}| = |\hat{v}_{1,p}| = |v_{2,p}| = |\hat{v}_{2,p}| = 1$. By means of the evaluation homomorphism ρ they can be lifted to modules of $U_q'\big(\widehat{sl}(2|2)\big)$. V is equipped with the following representation π_V: $U_q'\big(\widehat{sl}(2|2)\big) \to \mathrm{End}(V)$:

$$\begin{aligned}
&\rho\big(E_m^{1,+}\big)v_{0,p} = -q^{-2m(p-1)}[p]v_{2,p-1} && \rho\big(E_m^{1,-}\big)v_{0,p} = \rho\big(E_m^{1,-}\big)v_{1,p} = 0\\
&\rho\big(E_m^{1,+}\big)v_{1,p} = q^{-2mp}v_{3,p} && \rho\big(E_m^{1,-}\big)v_{2,p} = q^{-2mp}v_{0,p+1}\\
&\rho\big(E_m^{1,+}\big)v_{2,p} = \rho\big(E_m^{1,+}\big)v_{3,p} = 0 && \rho\big(E_m^{1,-}\big)v_{3,p} = -q^{-2mp}[p+1]v_{1,p}\\
&\rho\big(E_m^{2,+}\big)v_{0,p} = \rho\big(E_m^{2,+}\big)v_{1,p} = 0 && \rho\big(E_m^{2,-}\big)v_{0,p} = -q^{-m(2p-1)}[p+1]v_{2,p}\\
&\rho\big(E_m^{2,+}\big)v_{2,p} = q^{-m(2p-1)}v_{0,p} && \rho\big(E_m^{2,-}\big)v_{1,p} = q^{-m(2p+1)}v_{3,p+1}\\
&\rho\big(E_m^{2,+}\big)v_{3,p} = -q^{-m(2p-1)}[p]v_{1,p-1} && \rho\big(E_m^{2,-}\big)v_{2,p} = \rho\big(E_m^{2,-}\big)v_{3,p} = 0\\
&\rho\big(E_m^{3,+}\big)v_{0,p} = \rho\big(E_m^{3,+}\big)v_{2,p} = 0 && \rho\big(E_m^{3,-}\big)v_{0,p} = -q^{2m}[p]v_{1,p-1}\\
&\rho\big(E_m^{3,+}\big)v_{1,p} = -q^{2m}v_{0,p+1} && \rho\big(E_m^{3,-}\big)v_{2,p} = -q^{2m}v_{3,p}\\
&\rho\big(E_m^{3,+}\big)v_{3,p} = -q^{2m}[p+1]v_{2,p} && \rho\big(E_m^{3,-}\big)v_{1,p} = \rho\big(E_m^{3,-}\big)v_{3,p} = 0
\end{aligned} \tag{64}$$

and

$$\begin{aligned}
\rho\big(H_m^1\big)v_{k,p} &= -\rho\big(H_m^3\big)v_{k,p} = -\frac{1}{m}q^{m(p-2+\delta_{k,0})}\,[m(p+1-\delta_{k,0})]v_{k,p}\\
\rho\big(H_m^2\big)v_{k,p} &= -\frac{1}{m}q^{m(p-2+2\delta_{k,1}+\delta_{k,3})}\,[m(p+1-\delta_{k,3})]v_{k,p}.
\end{aligned} \tag{65}$$

Since $\pi_V(c) = 0$, the operator

$$R^{VV}(z) = (\pi_V \otimes \pi_V)\,\big(R(z)\big) \tag{66}$$

satisfies the Yang–Baxter equation

$$R_{12}^{VV}(z)R_{13}^{VV}(zw)R_{23}^{VV}(w) = R_{23}^{VV}(w)R_{13}^{VV}(zw)R_{12}^{VV}(z).$$

As in section 4, varying the action of H_n^4 on the module V just results in a change of a z-dependent factor common to all entries of the R-matrix. The action of H_0^4 on the modules introduced in the context of disordered systems [7] suggests the choice

$$H_n^4 v_{0,0} = -\frac{[n]}{n}v_{0,0} \qquad \text{for } n \neq 0. \tag{67}$$

Besides $R^{VV}(z)$, the models proposed in [7] involve the limit $q \to -1$ of $R^{\hat{V}V}(z)$ and $R^{V\hat{V}}(z)$. The relations of the corresponding R-matrices of the affine quantum super algebra among each other can be obtained by relating $\hat{V}$ to a module dual to V. The dual module V^* is introduced as the dual linear space to V with a $U_q'\big(\widehat{gl}(2|2)\big)$ structure given by

$$\langle a(v^*)|v\rangle = (-1)^{|a|\cdot|v^*|}\langle v^*|S(a)v\rangle \qquad a \in U_q'\big(\widehat{gl}(2|2)\big). \tag{68}$$

Let the dual basis be fixed by the canonical pairing $\langle v_{l,r}^*, v_{k,p}\rangle = \delta_{k,l}\delta_{p,r}$. Following [17] one finds that

$$\begin{aligned}
R^{V^*V}(z) &= \Big(\big(R^{VV}(z)\big)^{-1}\Big)^{T_1}\\
R^{VV^*}(z) &= \Big(\big(R^{VV}(z)\big)^{-1}\Big)^{T_2}\\
R^{V^*V^*}(z) &= \big(R^{VV}(z)\big)^{T_1T_2}.
\end{aligned} \tag{69}$$

Here T_1 and T_2 denote the graded transpositions over the first and second space, respectively. They are defined in terms of the usual transpositions by

$$R^{T_1} = R^{t_1} - 2\big((C - \mathrm{id}) \otimes \mathrm{id}\big)\, R^{t_1}\, \big(C \otimes \mathrm{id}\big)$$
$$R^{T_2} = R^{t_2} - 2\big(\mathrm{id} \otimes (C - \mathrm{id})\big)\, R^{t_2}\, \big(\mathrm{id} \otimes C\big) \tag{70}$$

where the map $C : V \to V$ is given by $C v_{k,p} = |v_{k,p}| v_{k,p}$. As a $U_q\big(gl(2|2)\big)$ module, V^* is related to the module $\hat{V}$ introduced in [22] by

$$v^*_{k,p} = \kappa_{k,p} \hat{v}_{k,p} \qquad \text{with } \kappa_{k,p} = (-1)^{p+\delta_{k,1}} q^{-(p+1)(p+|v_{k,p}|)} [p+1]^{|v_{k,p}|}. \tag{71}$$

Let matrix elements be assigned to $R^{VV}(z)$ according to

$$R^{VV}(z)\big(v_{k,p} \otimes v_{l,r}\big) = \sum_{k',l',p',r'} R^{k',p';\,l',r'}_{k,p;\,l,r}(z)\, v_{k',p'} \otimes v_{l',r'} \tag{72}$$

and analogously for the remaining entries. Then from (69) one obtains the relations

$$R^{\widehat{k',p'};\,l',r'}_{\widehat{k,p};\,l,r}\big(q^2 z^{-1}\big) = \frac{\kappa_{k',p'}}{\kappa_{k,p}} (-1)^{(|k'|-|k|)(|k|+1)} \big(R^{-1}\big)^{k,p;\,l',r'}_{k',p';\,l,r}(z^{-1})$$
$$R^{k',p';\,\widehat{l',r'}}_{k,p;\,\widehat{l,r}}\big(q^{-2} z^{-1}\big) = \frac{\kappa_{l',r'}}{\kappa_{l,r}} (-1)^{|l|(|l'|-|l|)} \big(R^{-1}\big)^{k',p';\,l,r}_{k,p;\,l',r'}(z^{-1}) \tag{73}$$
$$R^{\widehat{k',p'};\,\widehat{l',r'}}_{\widehat{k,p};\,\widehat{l,r}}(z) = \frac{\kappa_{k',p'}\kappa_{l',r'}}{\kappa_{k,p}\kappa_{l,r}} (-1)^{|k||l|+|k'||l'|} R^{k,p;\,l,r}_{k',p';\,l',r'}(z).$$

Explicit expressions for the above R-matrices, as well as a detailed description of the associated disordered systems, will be given elsewhere. Vertex models built from $R_{VV}(z)$ only may be studied by means of the algebraic Bethe ansatz. Ferromagnetic superspin chains involving the module V were considered in [11]. However, due to the non-integrability of the module V, the analysis of models accommodating both V and V^* requires much further investigation.

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References

[1] Nersesyan A A, Tsvelik A M and Wenger F 1994 Disorder effects in two-dimensional d-wave superconductors *Phys. Rev. Lett.* **72** 2628–31

Nersesyan A A, Tsvelik A M and Wenger F 1995 Disordered effects in two-dimensional Fermi systems with conical spectrum: exact results for the density of states *Nucl. Phys.* B **438** 561–88

Bernard D 1995 On the random vector potential in two dimensions *Nucl. Phys.* B **441** 471–82

Mudry C, Chamon C and Wen X-G 1996 Two-dimensional conformal field theory for disordered systems at criticality *Nucl. Phys.* B **466** 383–443

Chamon C, Mudry C and Wen X-G 1996 Instability of the disordered critical points of Dirac fermions *Phys. Rev.* B **53** R7638

Caux J S, Kogan I I and Tsvelik A M 1996 Logarithmic operators and hidden continuous symmetry in critical disordered models *Nucl. Phys.* B **466** 444

Kogan I I, Mudry C and Tsvelik A M 1996 Liouville theory as a model for prelocalized states in disordered conductors *Phys. Rev. Lett.* **77** 707

Castillo H E, Chamon C, Fradkin E, Goldbart P M and Mudry C 1997 Exact calculation of multifractal exponents of the critical wave function of Dirac fermions in a random magnetic field *Preprint* cond-mat/9706084

[2] Rozansky L and Saleur H 1992 Quantum field theory for the multi-variable Alexander-Conway polynomial *Nucl. Phys.* B **376** 461–509

[3] Dotsenko Vik S and Dotsenko Vl S 1983 Critical behavior of the phase transition in the 2D Ising model with impurities *Adv. Phys.* **32** 129–72

Shalaev B N 1994 Critical behavior of the two-dimensional Ising model with random bonds *Phys. Rep.* **237** 129–88

[4] Ludwig A M M, Fisher M P A, Shankar R and Grinstein G 1994 Integer quantum Hall transition: an alternative approach and exact results *Phys. Rev.* B **50** 7526–52

[5] Destri C and de Vega H J 1988 Integrable quantum field theories and conformal field theories from lattice models in the light-cone approach *Phys. Lett.* **201B** 261–68

Destri C and de Vega H J 1989 Light-cone lattices and the exact solution of chiral fermions and sigma models *J. Phys. A: Math. Gen.* **22** 1329–53

Bernard D 1995 (Perturbed) conformal field theory applied to 2d disordered systems: an introduction *Preprint* hep-th/9509137

[6] Cabra D C, Honecker A, Mussardo G and Pujol P 1997 A non-perturbative approach to the random-bond Ising model *Preprint* hep-th/9705180

[7] Zirnbauer M R 1997 Towards a theory of the integer quantum Hall transition: continuum limit of the Chalker–Coddington model *J. Math. Phys.* **38** 2007–36

[8] Jimbo M and Miwa T 1995 Algebraic analysis of solvable lattice models *CBMS Regional Conference Series in Mathematics* vol 85 (Providence, RI: American Mathematical Society)

[9] Bougourzi A H 1997 Bosonization of quantum affine groups and its applications to the higher spin model *Preprint* q-alg/9706015

[10] Korepin V E, Izergin A G, Essler F H L and Uglov D B 1994 Correlation function of the spin-$\frac{1}{2}$ XXX antiferromagnet *Phys. Lett.* **190A** 182–4

[11] Balents L, Fisher M P A and Zirnbauer M R 1996 Chiral Metal as a ferromagnetic super spin chain *Nucl. Phys.* B **483** 601

Gruzberg I A, Read N and Sachdev S 1997 Scaling and crossover functions for the conductance in the directed network model of edge states *Phys. Rev.* B **55** 10593

Gruzberg I A, Read N and Sachdev S 1997 Conductance and its universal fluctuations in the directed network model at the crossover to the quasi-one-dimensional regime *Preprint* cond-mat/9704032

[12] Bazhanov V V and Shadrikov A G 1987 Trigonometric solutions of triangle equations. Simple Lie superalgebras *Theor. Math. Phys. (USA)* **73** 1302–12

[13] Chang D, Phillips I and Rozansky L 1992 R-matrix approach to quantum superalgebra $su_q(m|n)$ *J. Math. Phys.* **33** 3710–5

[14] Zhang R B 1992 Universal L-operator and invariants of the quantum supergroup $U_q\big(gl(m|n)\big)$ *J. Math. Phys.* **33** 1970–9

[15] Jimbo M 1986 A q-analogue of $U_q\big(\widehat{gl}(N+1)\big)$, Hecke algebra, and the Yang–Baxter equation *J. Math. Phys.* **11** 247–52

[16] Zhang R B 1996 Symmetrizable quantum affine superalgebras and their representations *Preprint* q-alg/9612020

[17] Frenkel I and Reshetikhin N Yu 1992 Quantum affine algebras and holonomic difference equations *Commun. Math. Phys.* **146** 1–60

[18] Kimura K, Shiraishi J, Uchiyama J 1996 A level-one representation of the quantum affine superalgebra $U_q\big(\widehat{sl}(M+1|N+1)\big)$ *Preprint* q-alg/9605047

[19] Essler F H L and Korepin V E 1992 Higher conservation laws and algebraic Bethe Ansätze for the supersymmetric J–t model *Phys. Rev.* B **46** 9147–62

Foerster A and Karowski M 1993 The supersymmetric t–J model with quantum group invariance *Nucl. Phys.* B **408** 512–34

Maassarani Z 1995 $U_q osp(2,2)$ lattice models *J. Phys. A: Math. Gen.* **28** 1305–23

Bracken A J, Gould M D, Links J R and Zhang Y-Z 1995 New supersymmetric and exactly solvable model of correlated electrons *Phys. Rev. Lett.* **74** 2768–71

Pfannmüeller M P and Frahm H 1996 Algebraic Bethe ansatz for $gl(2|1)$ invariant vertex models *Preprint* cond-mat/9604082

[20] Drinfeld V G 1988 A new realization of Yangians and quantized affine algebras *Sov. Math. Dokl.* **36** 212–6

[21] Yamane H 1996 On defining relations of the Lie superalgebras and their quantized universal enveloping

superalgebras *Preprint* q-alg/9603015

[22] Gade R M 1997 On the quantum affine superalgebra $U_q\big(\widehat{gl}(2|2)\big)$ at level one *Nucl. Phys.* B **500** 547–64

[23] Zirnbauer M R 1994 Towards a theory of the integer quantum Hall transition: from the nonlinear sigma model to superspin chains *Ann. Phys., Lpz.* **3** 513–177

[24] Khoroshkin S M and Tolstoy V N 1991 Universal R-matrix for quantized (super)algebras *Commun. Math. Phys.* **141** 599–617

[25] Ding J and Frenkel I B 1993 Isomorphism of two realizations of quantum affine algebra $U_q\big(\widehat{gl}(n)\big)$ *Commun. Math. Phys.* **156** 277–300

Ding J and Iohara K Drinfeld comultiplication and vertex operators, RIMS-1091

[26] Zhang Y-Z 1997 Comments on Drinfeld realization of quantum affine superalgebra $U_q\big[gl(m|n)^{(1)}\big]$ and its Hopf algebra structure *Preprint* q-alg/9703020

[27] Ding J and Feigin B 1996 Difference equations of quantum current operators and parafermion construction *Preprint* q-alg/9610023

[28] Drinfeld V G 1986 Quantum groups *Proc. 1986 Int. Congr. of Mathematicians* ed A M Gleason (Berkeley, CA: American Mathematical Society)

[29] Khoroshkin S M and Tolstoy V N 1993 The Cartan–Weyl basis and the universal R-matrix for quantum Kac–Moody algebras and superalgebras *Proc. Int. Workshop on Mathematical Physics 'Quantum Symmetries' (2nd Wigner Symposium, Goslar, Germany, July 1991)* ed H-D Doebner and V K Dobrev, pp 336–51

[30] Drinfeld V G 1990 On almost cocommutative Hopf algebras *Leningrad Math. J.* **1** 321–42

[31] Jimbo M 1985 A q-analogue of $U\big(gl(N+1)\big)$, Hecke algebra, and the Young–Baxter equation *Lett. Math. Phys.* **10** 63–9

[32] Zhang R B 1992 Universal L-operator and invariants of the quantum supergroup $U_q\big(gl(m|n)\big)$ *J. Math. Phys.* **33** 1970–9

[33] Reshetikhin N Y and Semenov-Tian-Shansky M A 1990 Central extensions of quantum current groups *Lett. Math. Phys.* **19** 133–42

[34] Foda O, Iohara K, Jimbo M, Kedem R, Miwa T and Yan H 1995 Notes on highest weight modules of the elliptic algebra $\mathcal{A}_{q,p}\big(\widehat{sl}\big)$ *Prog. Theor. Phys. Suppl. (Japan)* **118** 1–34